Da Hyun Kim 74406 | 13:00 - 15:00 |

Exam I, MTH 213, Fall 2018

Ayman Badawi

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SCORE = ~~71~~ 65

QUESTION 1. (4 points) (SHOW THE STEPS) Find $(234)_8 \times (52)_8$

$$\begin{array}{r} (234)_8 \\ \times \quad (52)_8 \\ \hline (470)_8 \\ + (14140)_8 \\ \hline (14630)_8 \end{array}$$

$20 - 16 = 4$, $17 - 16 = 1$, $12 - 8 = 4$, $\frac{11}{8}$

$\therefore (234)_8 \times (52)_8 = (14630)_8$

QUESTION 2. (4 points) (SHOW THE STEPS) Convert 2078 to base 7.

$2078 \div 7 = 296$: $2078 - 14 \to 67 - 63 \to 48 - 42 = 6$

$296 \div 7 = 42$: $296 - 28 \to 16 - 14 = 2$

$42 \div 7 = 6$: $42 - 42 = 0$

$6 \div 7 = 0$ → stop: $6 - 0 = 6$

$2078 = (6026)_7$

QUESTION 3. (6 points) (SHOW STEPS) Let $d = gcd(121, 38)$. Find d, then find a, b such that $d = 121a + 38b$.

$121 = 3 \times 38 + 7$ ($121 - 114 = 7$)

$38 = 5 \times 7 + 3$ ($38 - 35 = 3$)

$7 = 2 \times 3 + 1$ ($7 - 6 = 1$)

$3 = 3 \times 1 + 0$ → stop; gcd ← 1

$\boxed{\therefore d = 1}$

$1 = 7 - 3 \times 2$

$1 = 7 - 2(38 - 5 \times 7)$

$1 = (121 - 3 \times 38) - 2(38 - 5(121 - 3 \times 38))$

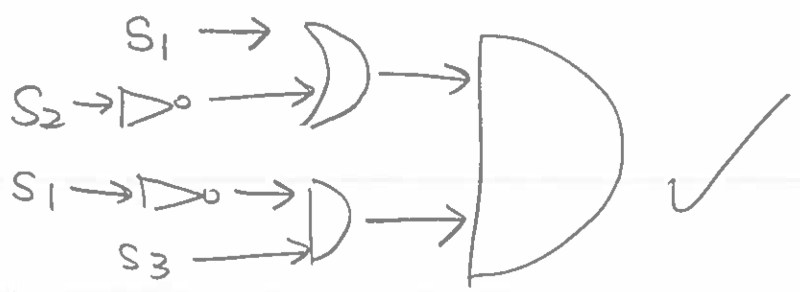
$1 = 121 - 3 \times 38 - 2(38 - 5 \times 121 + 15 \times 38)$

$1 = 121 - 3 \times 38 - 32 \times 38 + 10 \times 121$

$\therefore 1 = 11 \times 121 - 35 \times 38$

$\boxed{\therefore a = 11, \ b = -35, \ d = 1}$

QUESTION 4. (3 points) Draw circuit-gates that corresponds to the Boolean Algebra $(S_1 \vee \neg S_2) \wedge (\neg S_1 \wedge S_3)$ (i.e.,$(S_1 + \overline{S_2})\overline{S_1}S_3$).



QUESTION 5. (7 points) (SHOW STEPS) Let X be number of boys in a kiddy-garden. Given $X \equiv 2\ (mod\ 11)$, $X \equiv 1\ (mod\ 3)$, and $X \equiv 7\ (mod\ 8)$. Find X, where $0 < X < 264$.

$n_1 = 11,\ n_2 = 3,\ n_3 = 8,\ c_1 = 2,\ c_2 = 1,\ c_3 = 7$

gcd (of every two n_i's) $= 1 \Rightarrow \therefore$ CRT is applicable.

To find X: $m = n_1 \times n_2 \times n_3 = 11 \times 3 \times 8 = 264$

$m_1 = \frac{m}{n_1} = \frac{264}{11} = 24$ | $m_2 = \frac{m}{n_2} = \frac{264}{3} = 88$ | $m_3 = \frac{m}{n_3} = \frac{264}{8} = 33$

$24X\ (mod\ 11) = 1$ | $88X\ (mod\ 3) = 1$ | $33X\ (mod\ 8) = 1$

$2X\ (mod\ 11) = 1$ | $X\ (mod\ 3) = 1$ | $X\ (mod\ 8) = 1$

$\therefore X_1 = 6$ | $\therefore X_2 = 1$ | $\therefore X_3 = 1$

$X = (m_1 X_1 c_1 + m_2 X_2 c_2 + m_3 X_3 c_3)\ (mod\ m) = \big(24(6)(2) + (88)(1)(1) + 33(1)(7)\big)\ (mod\ 264)$

$\boxed{\therefore X = 79}$ ✓

QUESTION 6. (4 points) Assume that n^2 is even for some $n \in Z$. Use contradiction and convince me that n is an even integer.

n is an even integer

Deny $\Rightarrow$ n is an odd integer

Hence $n = 2k+1,\ k \in Z$.

$\underbrace{n^2}_{\text{even}} = (2k+1)^2 = 4k^2 + 4k + 1 = \underbrace{2(2k^2+2k)+1}_{\text{odd}}$ ✓

even = odd (x) $\Rightarrow$ contradiction.

The denial is invalid.

$\boxed{\therefore \text{ n is an even integer.}}$

QUESTION 7. (4 points) Convince me that $(\neg S_1 \wedge S_2) \vee (\neg S_2 \wedge S_1) \equiv S_1 \oplus S_2$ (i.e., $\overline{S_1}S_2 + \overline{S_2}S_1 \equiv S_1 \oplus S_2$).

S_1	S_2	$\neg S_1$	$\neg S_2$	$\neg S_1 \wedge S_2$	$\neg S_2 \wedge S_1$	$(\neg S_1 \wedge S_2) \vee (\neg S_2 \wedge S_1)$	$S_1 \oplus S_2$
0	0	1	1	0	0	0	0
0	1	1	0	1	0	1	1
1	0	0	1	0	1	1	1
1	1	0	0	0	0	0	0

identical ✓

$\boxed{\therefore (\neg S_1 \wedge S_2) \vee (\neg S_2 \wedge S_1) \equiv S_1 \oplus S_2}$

QUESTION 8. (5 points) Use the 4-method and convince me that $\sqrt{34}$ is irrational.

Deny $\sqrt{34}$ is rational.

$\sqrt{34} = \frac{a}{b}$ where $a, b \in Z$, $b \neq 0$, $gcd(a,b) = 1$

$34 = \frac{a^2}{b^2}$, a^2 is even $\rightarrow$ a is even $\rightarrow a = 2n,\ n \in Z$

b^2 is odd $\rightarrow$ b is odd. $\rightarrow b = 2m+1,\ m \in Z$

$34 = \frac{a^2}{b^2} = \frac{(2n)^2}{(2m+1)^2} = \frac{4n^2}{4m^2+4m+1} \Rightarrow 34(4m^2+4m+1) = 4n^2$

divide by 4

$34 \times 4m^2 + 34 \times 4m + 34 = 4n^2$ ✓

$\underbrace{34m^2 + 34m + 34/4}_{\text{not integer}} = \underbrace{n^2}_{\text{integer}} \Rightarrow$ contradiction.

Hence our denial is invalid. Therefore, $\sqrt{34}$ is irrational.

QUESTION 9. (6 points) Use Math Induction and convince me that $10 \mid n(n+1)(n+2)(n+3)(n+4)$ for every $n \geq 1$.

i) prove for n=1

$1(2)(3)(4)(5) = 120$

$\frac{120}{10} = 12$ ✓

∴ $10 \mid n(n+1)(n+2)(n+3)(n+4)$ for n=1

ii) Assume

$10 \mid n(n+1)(n+2)(n+3)(n+4)$ for some $n \geq 1$

iii) prove for n+1

Show $10 \mid (n+1)(n+2)(n+3)(n+4)(n+5)$.

$\frac{(n+1)(n+2)(n+3)(n+4)(n+5)}{10} = \frac{n(n+1)(n+2)(n+3)(n+4)}{10} + \frac{\cancel{5}(n+1)(n+2)(n+3)(n+4)}{\cancel{10}\ 2}$

↳ also integer ↳ integer ↳ integer

integer + integer = integer ∵ step(ii)

∵ odd × even × odd × even is even

$\Rightarrow 2a(2b+1)(2c)(2d+1)$

$= (4ab+2a)(4cd+2c)$

$= 16abcd + 8abc + 8acd + 4ac$

$= 2(8abcd + 4abc + 4acd + 2ac)$

where $a,b,c,d \in \mathbb{Z}$

$\boxed{\therefore 10 \mid n(n+1)(n+2)(n+3)(n+4) \text{ for every } n \geq 1}$ ✓

QUESTION 10. (6 points) Write down T or F

(i) $x^2 + 4 = -8$ for some $x \in Z$ if and only if $x^2 + 5 = -7$ for some $x \in Z$. $\boxed{T}$ ✓ (False ... iff ... False)

(ii) $\exists!\, a \in Q$ such that $ab \in Z, \forall b \in Q$. $\boxed{F}$

(iii) If $x^4 = 16$ for some $x \in Q$, then $x + 4 = 6$. $\boxed{F}$ when x=-2 it's false ✓ ($x=2$ and $x=-2$)

(iv) $\{(1,2),(2,1),(1,1)\}$ is an equivalence relation on the set $A = \{1,2\}$. $\boxed{T}$ ↳ subset of A.

(v) $0101 \oplus 1010 = 1111$ $\boxed{T}$ ✓

(vi) For every $a \in Q^*$, $\exists b \in N^*$ such that $a^3 = b$. $\boxed{F}$ ✓

QUESTION 11. (6 points) Let $A = \{1, 2, 3, 4, 5, 6\}$ and ~~$B = \{1, 6\}$~~ $B = \{1, 4, 6\}$. Define " = " on A such that $\forall a, b \in A$, a" = "b iff $ab(mod 7) \in B$. Then "=" is an equivalence relation on A (do not show that).

Find all equivalence classes of " = ".

$[1] = \{1, 6\}$

$[2] = \{2,$

How many elements does " = " have?

Is $(3,6) \in$"="? Why?

Is 3"="6 ?

Is $(4,5) \in$"="? Why?

Is 4"="5?

QUESTION 12. (i) (3 points) What is 5^{1002} $(mod\ 11)$?

$\gcd(5, 11) = 1$, $\phi(11) = 10$

By Euler-Fermat Theorem, $5^{10} \pmod{11} = 1$

$\Rightarrow (5^{10})^{100} \pmod{11} = 1^{100} \Rightarrow 5^{1000} \pmod{11} = 1$ ✓

$5^{1000+2} \pmod{11} = 5^{1000} \cdot 5^2 \pmod{11} = 3$

$\boxed{\therefore 5^{1002} \pmod{11} = 3}$

(ii) **(4 points)**Find all possible values of X over Planet Z_{12} where $9X = 6$.

$9X=6$ in Z_{12}

$a=9, n=12$

$gcd(9,12)=3$

Is $3|6$? yes

Therefore, there must be 3 different solutions.

To find X_1, try and error.

$9X \pmod{12} = 6 \rightarrow \therefore X_1 = 2$

$n = 12 = 3 \times \boxed{4}$

Set of solutions: $\{2, 6, 10\}$

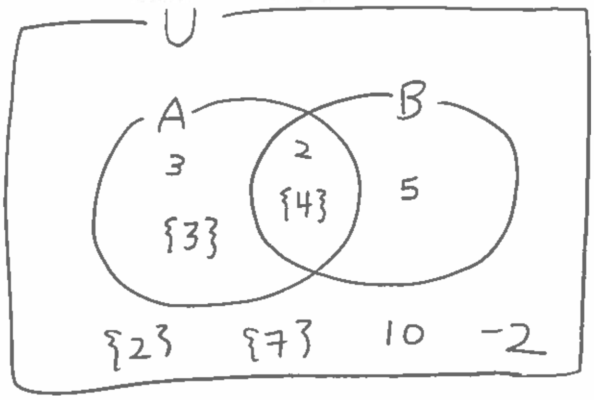
(iii) **(2 points)**Find all possible values of X over Planet Z where $9X \equiv 6 \ (mod\ 12)$.

the answer is $2+4k$, $k \in Z$.

QUESTION 13. **(7 points)** Let $A = \{3, \{4\}, \{3\}, 2\}$, $B = \{2, \{4\}, 5\}$, and $U = \{3, \{3\}, 2, \{2\}, \{4\}, 5, \{7\}, 10, -2\}$. Then

(i) Find $A - B$.

$A-B = \{3, \{3\}\}$



(ii) Find $\overline{A} \cap B$

$B-A = \{5\}$

(iii) True of F

- $\{\{3\}, 2\} \subset A$ T
- $\{4\} \subset B$. F
- $\{7\} \subset \overline{B}$ F
- $\{3, 2\} \subset P(A)$. F
- $\{(3, \{3\})\} \subset P(A \times U)$. T

$3, 2 \in P(A)$

$3, 2 \subset A$

$\{(3, \{3\})\} \in P(A \times U)$

Faculty information

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